

In complete sentences, using proper English and mathematical notation,

SCORE: \_\_\_\_ / 5 PTS

state the Fundamental Theorem of Calculus (both parts) and the Net Change Theorem.

FTC 1: If  $f$  is continuous on an interval containing  $a$ , and  $g(x) = \int_a^x f(t) dt$ ,

then  $g'(x) = f(x)$  for all  $x$  on that interval.

FTC 2: If  $f$  is continuous on  $[a, b]$ , and  $F' = f$  on  $[a, b]$ ,

then  $\int_a^b f(x) dx = F(b) - F(a)$ .

NCT: If  $F'$  is continuous on  $[a, b]$ ,

then  $\int_a^b F'(x) dx = F(b) - F(a)$ .

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Let  $\rho(x)$  be the linear density of a rod (in grams per centimeter) at the point  $x$  centimeters from the left end      SCORE: \_\_\_\_ / 2 PTS  
(eg.  $\rho(12) = 10$  means that at the point 12 cm from the left end, the rod's density is 10 grams per centimeter of rod).

What is the meaning of the equation  $\int_{50}^{60} \rho(x) dx = 80$  ?

**NOTES:**      Your answer must use all three numbers from the equation, along with correct units.  
Your answer should NOT use “x”, “ $\rho$ ”, “integral”, “antiderivative”, “rate of change”, “change” or “derivative”.  
Your answer should sound like normal spoken English.

The part of the rod between 50 cm from the left end to 60 cm from the left end  
has a mass of 80 grams

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Evaluate the following integrals, or explain why they can't be evaluated.

SCORE: \_\_\_\_ / 12 PTS

[a]  $\int_{-1}^1 \frac{\tan y}{1 + \sin^2 y} dy$

①  $\frac{\tan(-y)}{1 + \sin^2(-y)} = \frac{-\tan y}{1 + (-\sin y)^2} = -\frac{\tan y}{1 + \sin^2 y}$

Integrand is odd and continuous, so integral = 0

①/2

①/2

[b]  $\int_{-3}^3 \frac{2t^3}{t^4 - 16} dt$

Integrand is not continuous (vertical asymptote) @  $t = \pm 2$  ①

so FTC2 does not apply

①/2

[c]  $\int_2^4 \frac{5x^2}{16 + 4x^6} dx$

$= \frac{5}{16} \int_2^4 \frac{x^2}{1 + \frac{1}{4}x^6} dx$

①  $u = \frac{1}{2}x^3 \Rightarrow \frac{du}{dx} = \frac{3}{2}x^2 \Rightarrow \frac{2}{3}du = x^2 dx$

$x = 4 \Rightarrow u = 32$

$x = 2 \Rightarrow u = 4$

①  $\frac{5}{16} \cdot \frac{2}{3} \int_4^{32} \frac{1}{1 + u^2} du$  ①/2

①  $= \frac{5}{24} \arctan u \Big|_4^{32}$  ①/2

$= \frac{5}{24} (\arctan 32 - \arctan 4)$  ①/2

[d]  $\int \frac{(3z^2 - 2\sqrt{z})^2}{18z^5} dz$

$= \int \frac{9z^4 - 12z^{\frac{5}{2}} + 4z}{18z^5} dz$

$= \int \left( \frac{1}{2}z^{-1} - \frac{2}{3}z^{-\frac{5}{2}} + \frac{2}{9}z^{-4} \right) dz$  ①

$= \frac{1}{2} \ln |z| + \frac{4}{9} z^{-\frac{3}{2}} - \frac{2}{27} z^{-3} + C$  ①/2

①

①



If  $b(x) = \int_{3^x}^{x^3} \arctan(1+t^2) dt$ , find  $b'(x)$ .

SCORE: \_\_\_\_ / 4 PTS

$$b(x) = \int_{3^x}^0 \arctan(1+t^2) dt + \int_0^{x^3} \arctan(1+t^2) dt = -\int_0^{3^x} \arctan(1+t^2) dt + \int_0^{x^3} \arctan(1+t^2) dt$$

$$b'(x) = -\frac{d}{dx} \int_0^{3^x} \arctan(1+t^2) dt + \frac{d}{dx} \int_0^{x^3} \arctan(1+t^2) dt$$

$$b'(x) = -\left( \frac{d}{d(3^x)} \int_0^{3^x} \arctan(1+t^2) dt \right) \frac{d(3^x)}{dx} + \left( \frac{d}{d(x^3)} \int_0^{x^3} \arctan(1+t^2) dt \right) \frac{d(x^3)}{dx}$$

$$b'(x) = -\left( \arctan(1+(3^x)^2) \right) (3^x \ln 3) + \left( \arctan(1+(x^3)^2) \right) (3x^2)$$

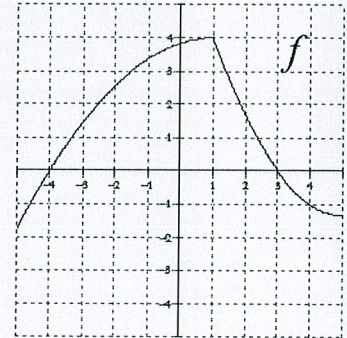
$$b'(x) = \boxed{-3^x (\ln 3) \arctan(1+3^{2x})} + \boxed{3x^2 \arctan(1+x^6)}$$



Let  $p(x) = \int_{-2}^x f(t) dt$ , where  $f$  is the function whose graph is shown on the right.

SCORE: \_\_\_\_ / 7 PTS

[a] Write "I UNDERSTAND" to indicate that you understand that the graph shows  $f$ , but that the questions below ask about  $p$ .



[b] Find  $p'(1)$ . Explain your answer very briefly.

$$\boxed{p'(1) = f(1) = 4}$$

(1)      (1)

[c] Find all intervals over which  $p$  is both decreasing and concave up at the same time. Explain your answer very briefly.

$$\boxed{p'(x) = f(x) \text{ is negative and increasing on } [-5, -4]}$$

(2)                      (1)

[d] Find all critical numbers of  $p$ . Explain your answer very briefly.

$$\boxed{p'(x) = f(x) = 0 \text{ at } x = -4, 3}$$

(1)                      (1)

(There are no critical numbers corresponding to  $p'(x)$  DNE)